A Self-Calibrated Method to Measure the Load Coefficient of the Resistor

Z. Li, Q. He, and Z. Zhang

National Institute of Metrology, China

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1. Introduction

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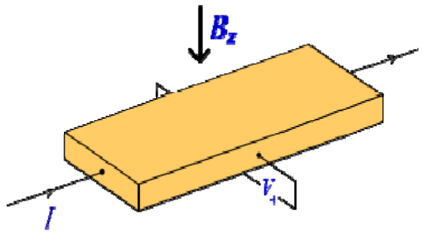
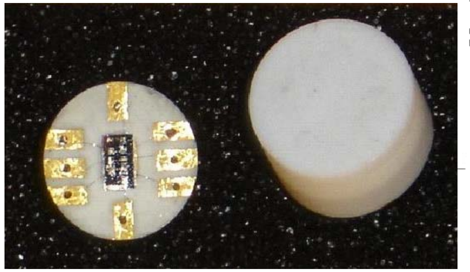
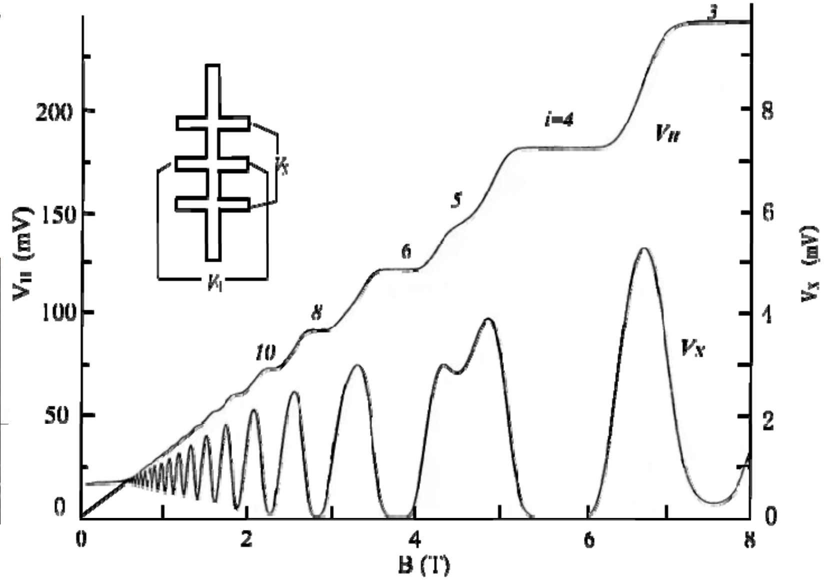






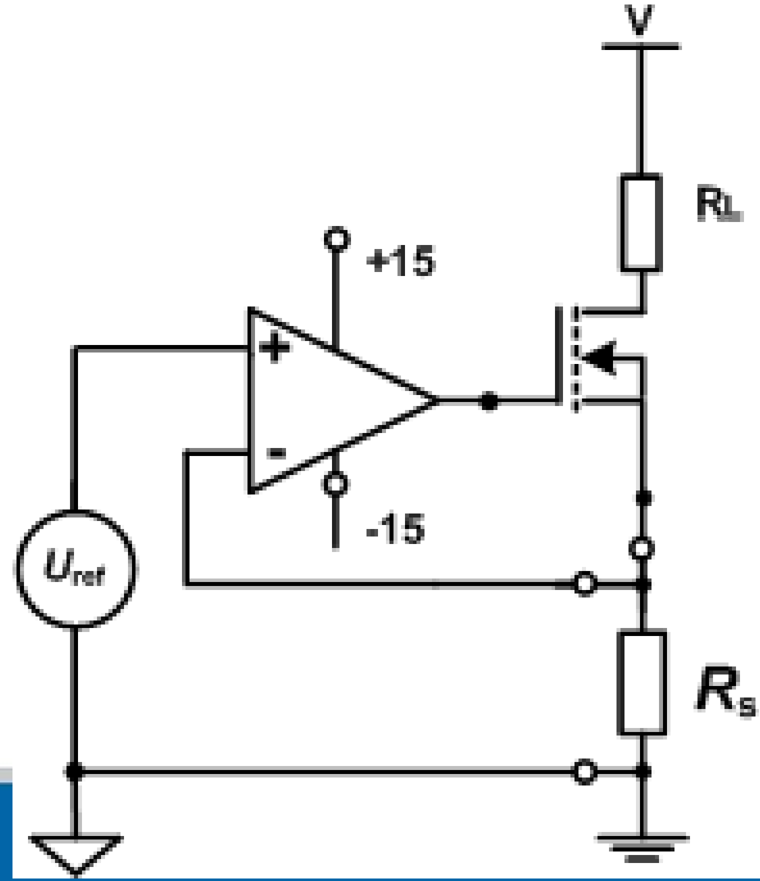
$$R_H = \frac{V_H}{I} = \frac{h}{e^2 i} = \frac{25812.807}{i}\Omega \qquad i = 1,2,3\ldots$$

- Uncertainty: $10^{-6} \sim 10^{-7}$ -> $10^{-9} \sim 10^{-10}$

- In metrology laboratory, The value of the resistor can be traced to the QHR with
  - Cryogenic Current Comparator (CCC) bridge
  - Direct Current Comparator (DCC) bridge.
- In international comparison and calibration, the currents are set at some certain values,
  - QHR(i=2) @38.7uA : 100ohm @5mA
  - 100 ohm@0.5mA : 1ohm @50mA

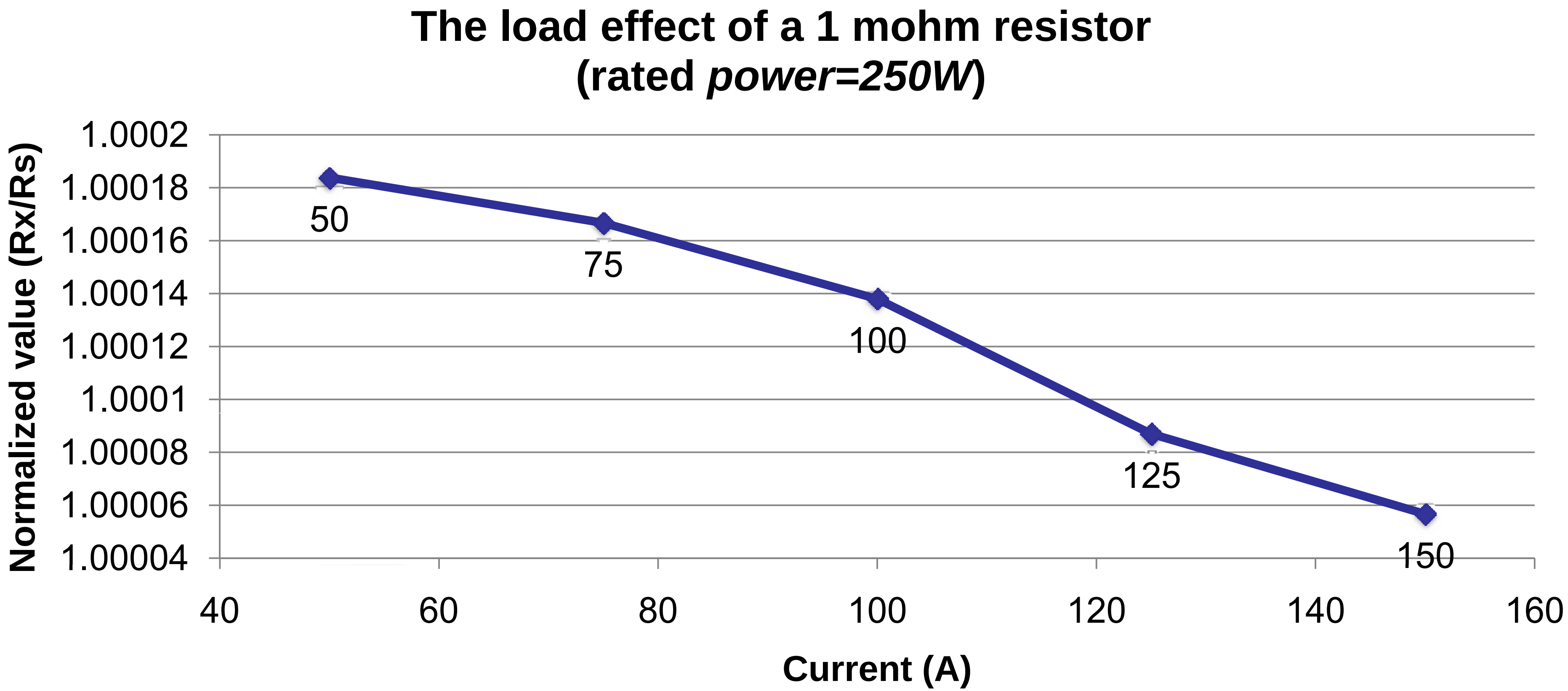
The problem from load effect

- Moreover, in industry and research field, the resistors with different values work in a wide range of current.
  - Current source,
  - Current sampling
  - Current sensor, etc.



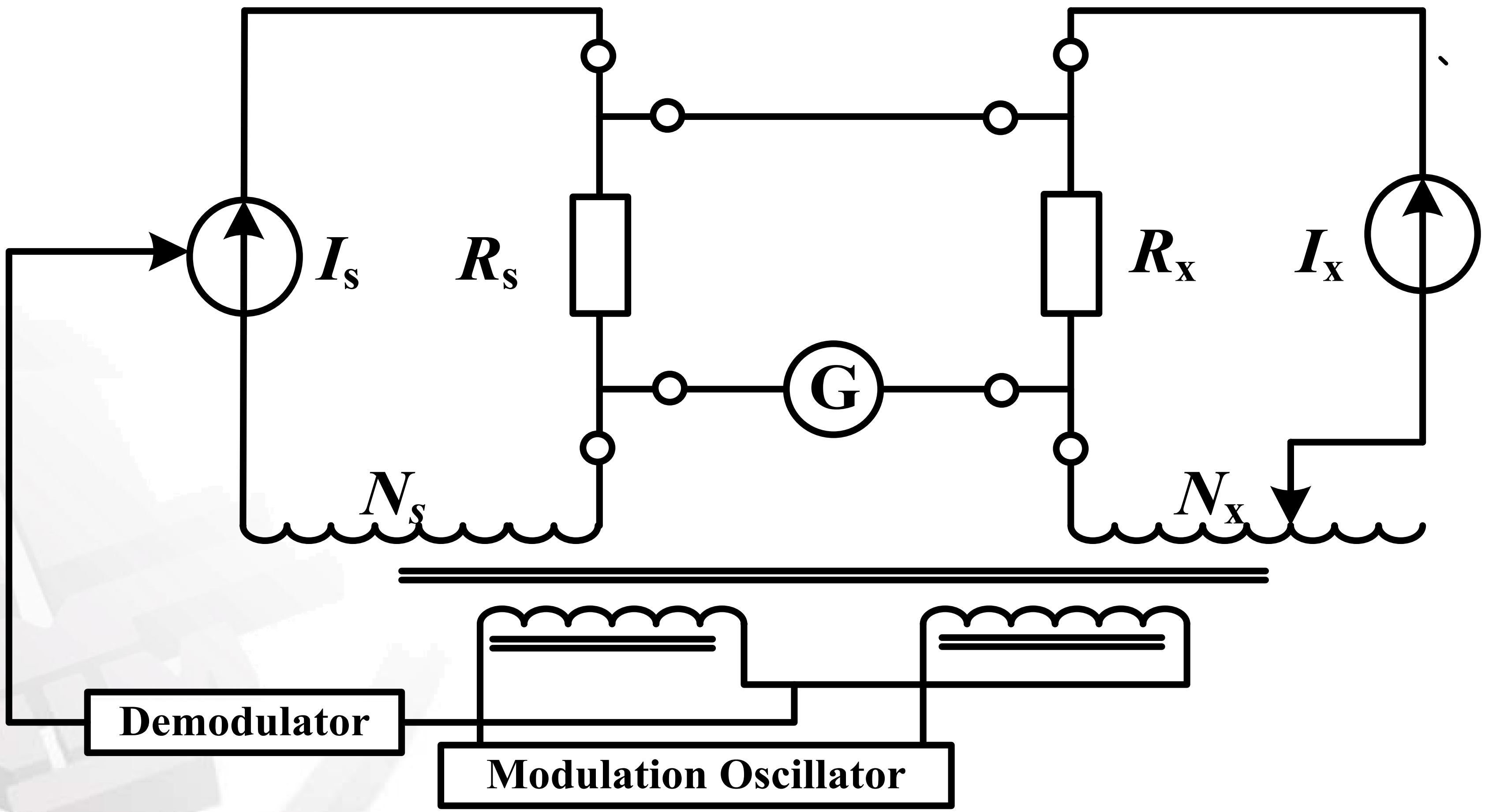
- The value of the resistor has a dependence on the current through it.
  - Current source:

$$I = \frac{U_{ref}}{R}$$



- The load coefficient has to be calibrated and can be further used to correct the output of the current source.

- The reasons for the load effect
  - Self-heating from current through it
  - The heat resistance around the resistor
  - The environment temperature.


Is
Rs
Rx
Ix
G
Ns
Nx
Demodulator
Modulation Oscillator

- With the DCC bridge, the ratios between two resistors can be got.

$$\gamma_1 = \frac{R_X}{R_S} = \frac{R_{X0}(1+\alpha_1)}{R_{S0}(1+\beta_1)}$$

- When the voltages on both resistors are increased *N* times,

$$\gamma_2 = \frac{R_X^{'}}{R_S^{'}} = \frac{R_{X0}(1+\alpha_2)}{R_{S0}(1+\beta_2)}$$

$$\frac{\gamma_2}{\gamma_1} = \frac{\dfrac{R_{X0}(1+\alpha_2)}{R_{S0}(1+\beta_2)}}{\dfrac{R_{X0}(1+\alpha_1)}{R_{S0}(1+\beta_1)}} \approx 1+(\alpha_2-\alpha_1)+(\beta_1-\beta_2)$$

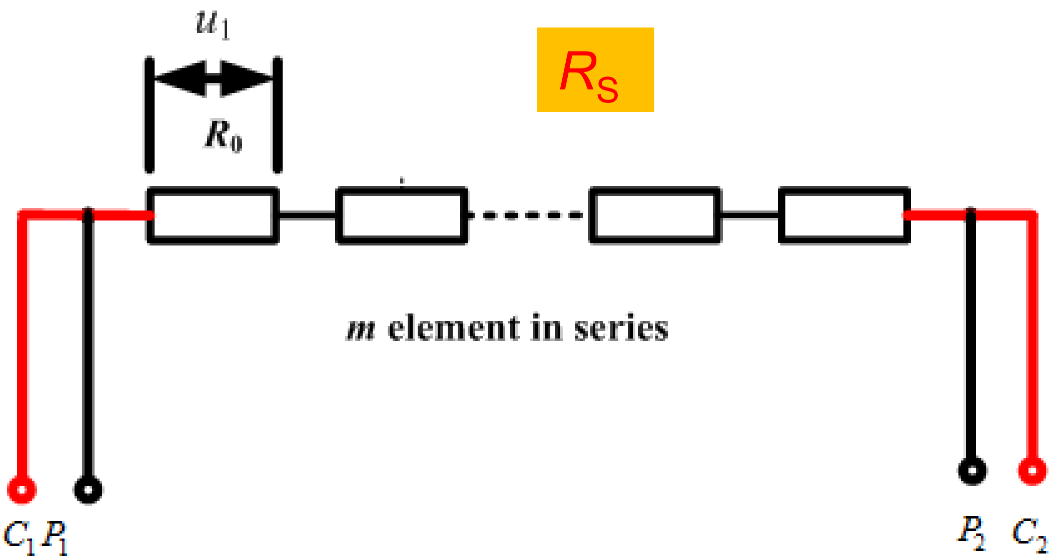
- When the rated power of R_S is high enough and temperature coefficient is low enough, the change of R_S could be very small.

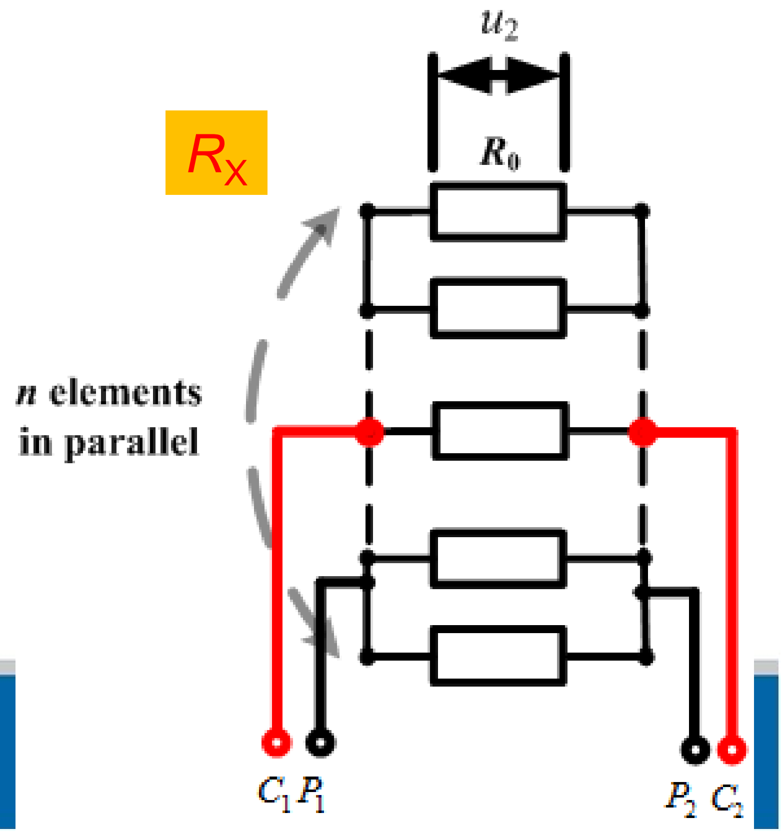
  i.e. β_2-$\beta_1 \approx 0$. $\frac{\gamma_2}{\gamma_1} \approx 1+(\alpha_2-\alpha_1)$

But in metrology, we need to know how small? How small is enough? i.e. a reference is needed.

2. The principle

- Resistors with a same type and almost the same temperature coefficient should be selected to form following two combined resistors, R_S and R_X





- When they are compared with a DCC bridge,

$$\frac{u_2}{u_1} = m$$

$$R_X = R_{X0}(1+\alpha) \qquad R_S = R_{S0}(1+\beta)$$

- R_S and R_X are formed with the same type elements, thus we can have

$$\alpha \propto u_2^2, \qquad \beta \propto u_1^2, \qquad \Rightarrow \frac{\alpha}{\beta} = m^2$$

- Increase the current on R_X k times, the current on R_S will also increase **almost** ***k*** times. But the ratio of voltages on the elements of two resistors will be still the same as

$$\frac{u_2'}{u_1'} = \frac{ku_2}{ku_1} = m$$

- R_S and R_X become,

$$R_x' = R_{X0}(1 + k^2\alpha) \qquad R_S' = R_{S0}(1 + k^2\beta)$$

- In above two cases, we can get

$$\gamma = \frac{R_X}{R_S} = \frac{R_{X0}(1+\alpha)}{R_{S0}(1+\beta)}$$

$$\gamma' = \frac{R_X'}{R_S'} = \frac{R_{X0}(1+k^2\alpha)}{R_{S0}(1+k^2\beta)}$$

$$\frac{\gamma'}{\gamma} = \frac{\dfrac{R_{X0}(1+k^2\alpha)}{R_{S0}(1+k^2\beta)}}{\dfrac{R_{X0}(1+\alpha)}{R_{S0}(1+\beta)}} \approx \frac{1+k^2(\alpha-\beta)}{1+(\alpha-\beta)}$$

- We already know that

$$\alpha \propto u_2^2, \qquad \beta \propto u_1^2, \qquad \Rightarrow \frac{\alpha}{\beta} = m^2$$

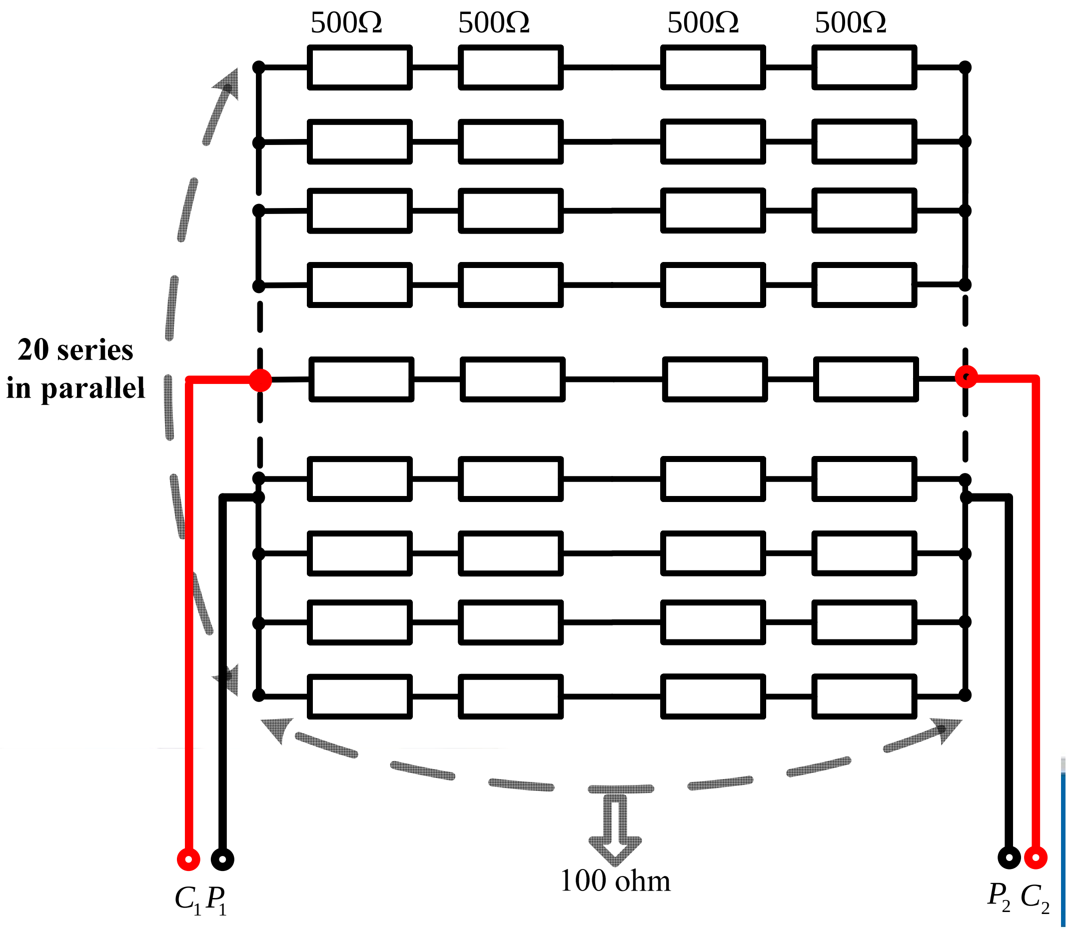
- Thus, it can be got

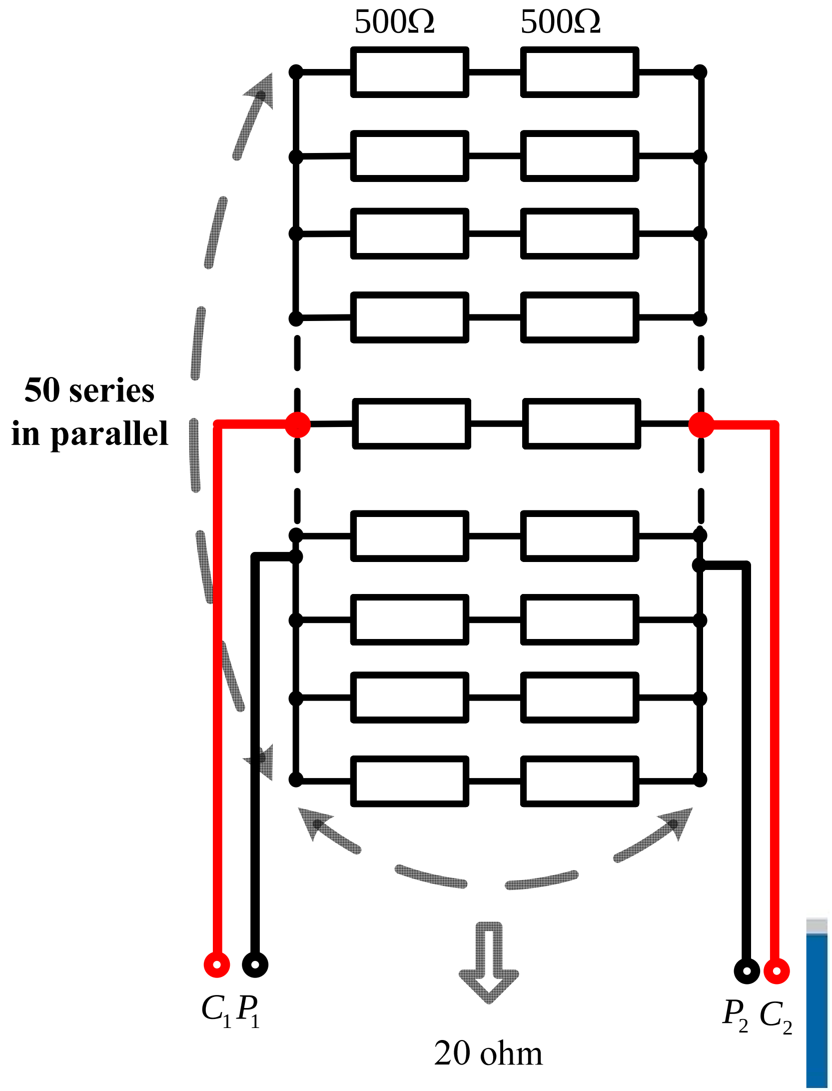
$$\frac{\gamma'}{\gamma} \approx \frac{1+k^2(\alpha-\beta)}{1+(\alpha-\beta)} = 1+\beta(1-k^2)(1-m^2)$$

$$\begin{cases} \beta = \dfrac{\dfrac{\gamma'}{\gamma}-1}{(1-k^2)(1-m^2)} \\ \alpha = m^2\beta \end{cases}$$

3. Measurement results

- The voltage on resistor is set as 0.5V and 1V
- *m*=2,*k*=2





- K=2, m=2

$$\begin{cases} \beta = \frac{1}{9}\left(\frac{\gamma'}{\gamma} - 1\right) \\ \alpha = m^2 \beta = \frac{4}{9}\left(\frac{\gamma'}{\gamma} - 1\right) \end{cases}$$

Temperature of the oil bath / °C	The voltage on two resistors / V	Average of the ratio (γ)	Load coefficient / ohm / ohm
23	0.5	5.00002229	α=-1.7E-08 β=-4.2E-09
	1	5.00002210	

$$\frac{\gamma_2}{\gamma_1}=\frac{\dfrac{R_{X0}(1+\alpha_2)}{R_{S0}(1+\beta_2)}}{\dfrac{R_{X0}(1+\alpha_1)}{R_{S0}(1+\beta_1)}}\approx 1+(\alpha_2-\alpha_1)+(\beta_1-\beta_2)$$

- When the rated power of R_S is high enough and temperature coefficient is low enough, the change of R_S could be very small.

i.e. β_2-$\beta_1 \approx 0$. $\quad \frac{\gamma_2}{\gamma_1} \approx 1+(\alpha_2-\alpha_1)$

But in metrology, we need to know how small? How small is enough? i.e. a reference is needed.

- With calibrated resistor as reference, other resistor's load coefficient can be calibrated. Here, β_1-β_2 is known, then we can get,

$$(\alpha_2 - \alpha_1) = \frac{\gamma_2}{\gamma_1} - 1 - (\beta_1 - \beta_2)$$

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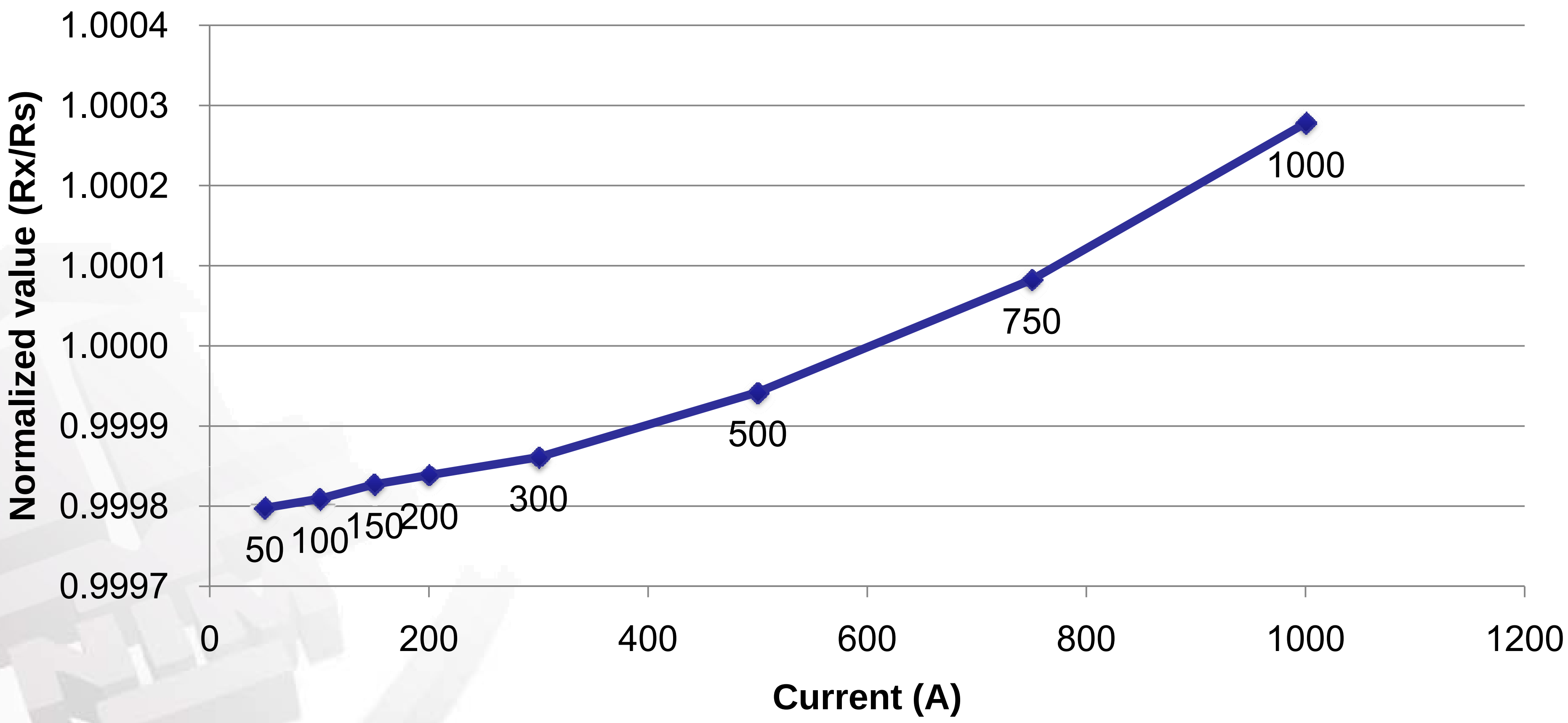
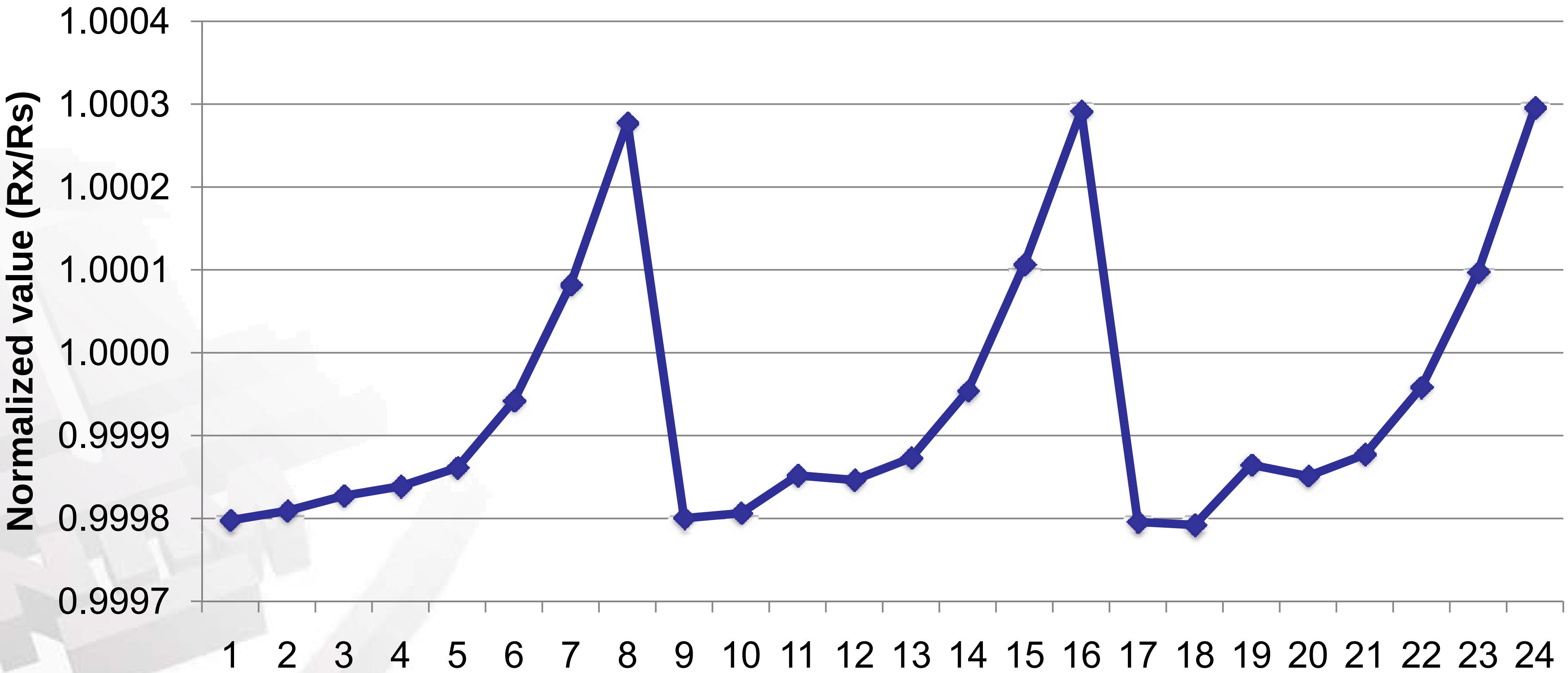
The load effect of a 1000A DC Shunt (0.1 mohm)



Calibration of the Load effect of Guildline 1000A Shunt
（reference: 20 ohm lab-made low load effect resistor）



Key point for self-calibration

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- Resistor elements should be the same type and have same temperature coefficients.
- The resolution and short term stability, repeatability of the DCC bridge is very important for this method. So the DCC bridge can be not be rebooted in the measurement procedure.

4. Conclusion

- A self-calibrated method to measure the load coefficient of resistors is proposed. The absolute load coefficient of both resistors can be got with this approach.

- Further, the calibrated resistors can be used to calibrate other resistor.

Thank you for your attention!

Question?